Are Post-Glacial Rebound Model Predictions Consistent with the Global Space-Geodetic Secular Velocity Field ?

Corné Kreemer, Hans-Peter Plag – Nevada Geodetic Laboratory, Nevada Bureau of Mines and Geology, University of Nevada, Reno
David Lavallée – University of Newcastle upon Tyne, U.K.

Summary

Many global post-glacial rebound (PGR) velocity models currently exist. None of these 3-D models have been validated on a global scale using space-geodetic data. This could be done directly using the vertical velocities if one considers any possible reference frame difference between the model and observations: $\bar{v}_{up}^{pgr} = \alpha \bar{v}_{up}^{obs} + \overline{X}$, where α is a scale and $\overline{X}$ is the translation rate of the origin. Here we are particularly interested in the horizontal velocity field. Thus, once placed in a similar frame, the horizontal PGR velocity v_{hor}^{pgr} has to be separated from rigid body rotation describing plate motion, where a scalar coefficient γ could be introduced to scale the PGR prediction if needed: $\bar{v}_{hor}^{obs,translated} = \gamma \bar{v}_{hor}^{pgr} + [\overline{\Omega} \times \bar{r}]$, where Ω is the angular velocity vector and $\bar{r}$ the position vector.

Here we use our own global GPS velocity solution. This study allows us to test the consistency and validity of several PGR models, and also to investigate if and how rigid body plate rotation estimates improve by subtracting a PGR signal from the observations.

We find that there to be a significant difference between the ITRF2000 reference frame and those of the PGR models. The difference is reflected in α being 1-2 and the norm of $\overline{X}$ to be between 1.2-2.1 mm/yr. We also find that there is a large variance between the different PGR models, particularly in the horizontal velocity predictions, and this leads to varying results in whether and how the consideration of a PGR signal in estimating a rigid body rotation can improve the fit to the data. The consideration of all PGR models leads to an improved fit for Eurasia when solving for $\overline{\Omega}$ and γ, with γ varying between 0.5-1.5 for the different models. For North America the consideration of the REF,ALT and JXM models can lead to an improved fit (γ between 0.5-1.4), but the VM models are inconsistent with the observations. We also observed for some models statistical improvements for South America and Australia, although for the latter a negative γ is required, suggesting that the PGR models may have the wrong sign. It still needs to be tested whether the obtained angular velocities for various plates and models are significantly different or not.

PGR Model Predictions

Predictions of the secular PGR signal in surface displacement are taken from the Special Bureau for Loading (SBL) of the IERS. The available predictions are:

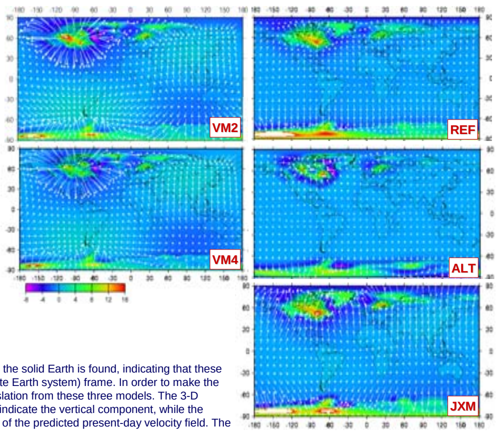
Model description:

Model	Author	Ice	Earth
VM2	Peltier (2004,2005)	ICE-5G V2	Depth-dependent parameters, 90 km lithosphere
VM4	Peltier (2004,2005)	ICE-5G V2	Same as VM2 but lower viscosity in upper mantle
REF	Schotman et al. (2005)	ICE-3G	5 homogeneous layers, 98 km lithosphere, higher viscosity in lower mantle
ALT	Schotman et al. (2005)	mod. ICE-3G	Same as REF, but homogeneous viscosity in mantle
JXM	Mitrovica (Milne et al., 1999)	ICE-3G	4 homogeneous layers, 120 km lithosphere, high viscosity in lower mantle

For all predictions, the solid Earth models are spherically symmetric and incompressible. For predictions given in the CE (center of mass of the solid Earth) frame, the predictions should satisfy a no-net-translation (NNT) condition for the solid Earth. In order to test this condition, we computed the global means (with areal weighing) for each component, which should be zero in the case of NNT:

Mean velocities in mm/yr

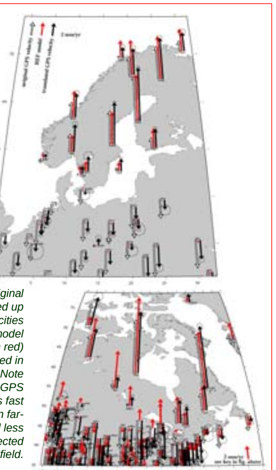
Model	East	North	Up	Norm
VM4	$4.2 \cdot 10^{-5}$	0.30	$-4.4 \cdot 10^{-2}$	1.22
VM2	$-3.2 \cdot 10^{-6}$	0.39	$-6.3 \cdot 10^{-3}$	1.43
REF	$1.4 \cdot 10^{-5}$	$-2.3 \cdot 10^{-2}$	$1.6 \cdot 10^{-4}$	0.98
ALT	$1.5 \cdot 10^{-5}$	$-3.3 \cdot 10^{-2}$	$8.0 \cdot 10^{-6}$	0.51
JXM	$7.3 \cdot 10^{-6}$	0.37	$3.2 \cdot 10^{-5}$	1.54



For VM2, VM4, and JXM, a significant northward translation of the solid Earth is found, indicating that these predictions are given in the CM (Center of Mass of the complete Earth system) frame. In order to make the predictions comparable, we have removed the northward translation from these three models. The 3-D velocity fields shown on the right are for the CE frame. Colors indicate the vertical component, while the arrows give the direction and size of the horizontal component of the predicted present-day velocity field. The scales for all models are identical.

GPS Velocity Model

Our GPS solution consists of a combination of weekly global and regional GPS solutions for 376 stations between 1999 and 2005. Weekly station coordinate estimates from the Scripps global IGS analysis center and 5 regional associate analysis centers (Australia, Europe, Japan, and North- and South America) are rigorously combined using a free-network approach [*Davies and Blewitt*, 2000]. A modified Helmert blocking approach is taken utilizing stochastic modeling to minimize frame bias. Weekly evolving variance component estimates, antenna height corrections and a three-dimensional data-snooping outlier rejection method are also applied. Any stations appearing in a minimum of 104 observations over a minimum of 2.5 year data-span are fitted to a constant linear station motion model applying minimal constraints for network orientation and orientation rate. The resulting free network solution is aligned to ITRF2000 by estimating a 12 parameter Helmert transformation, this infers an origin from satellite laser ranging; i.e., free from GPS orbit modeling.



Examples original and translated up GPS velocities with REF model (shown in red) used in translation. Note how original GPS data shows fast subsidence in far-field and less than expected uplift in near-field.

Frame Adjustment

In order to use the geodetic velocities in a study of the PGR models, the observed velocities need to be placed in a similar reference frame as the PGR predictions. We do this for each PGR model separately by calculating a scale and translation rate from a least square fit of the 220 vertical velocities for sites on 15 tectonic plates. The results are tabulated below.

Model	α (scale)	$\dot{X}_x$ mm yr^{-1}	$\dot{X}_y$ mm yr^{-1}	$\dot{X}_z$ mm yr^{-1}
VM2	1.860 ± 0.009	1.698 ± 0.042	-0.146 ± 0.022	1.224 ± 0.032
VM4	1.385 ± 0.013	1.345 ± 0.043	-0.142 ± 0.022	0.689 ± 0.032
REF	1.145 ± 0.013	0.904 ± 0.042	-0.315 ± 0.022	1.409 ± 0.033
ALT	0.978 ± 0.017	0.769 ± 0.042	-0.041 ± 0.022	0.958 ± 0.032
JXM	1.441 ± 0.010	0.906 ± 0.042	-0.273 ± 0.022	1.513 ± 0.032

All models suggest a translation of the GPS velocities of ~1.2-2.1 mm/yr towards western Europe, and a scale change of a factor between 1 and 2.

PGR Model Inter-Comparison

Horizontal displacement vectors:

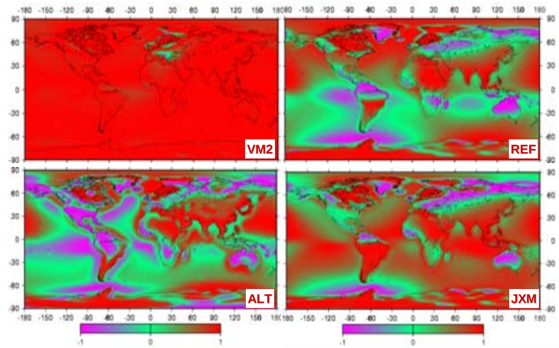
	VM2	VM4	REF	ALT	JXM
VM2	1.000	0.947	0.279	-0.252	0.368
VM4	0.947	1.000	0.124	-0.412	0.132
REF	0.279	0.124	1.000	0.551	0.811
ALT	-0.252	-0.412	0.551	1.000	0.673
JXM	0.368	0.132	0.811	0.673	1.000

3-D displacement vectors:

	VM2	VM4	REF	ALT	JXM
VM2	1.000	0.955	0.637	0.447	0.712
VM4	0.955	1.000	0.504	0.382	0.580
REF	0.637	0.504	1.000	0.652	0.929
ALT	0.447	0.382	0.652	1.000	0.679
JXM	0.712	0.580	0.929	0.679	1.000

For the inter-comparison of the models we have considered, among others, cross correlation of the individual components of the predicted velocities as well as the horizontal and total vectors, and the spatial pattern of the scalar product of pairs of predictions. For any pair of models, global spatial correlation coefficients for the individual horizontal components are generally much lower than for the up component, and for ALT they are negative for VM2 and VM4. In the table on the left we give the correlation coefficients for the horizontal and total (3-D) velocity vectors, respectively.

For the pair (VM2,VM4) correlation is very high (above 0.9 for all components and vectors), indicating a high consistency of these two models. A rather high correlation is also found for the pair (REF,JXM). Lowest correlation is found between ALT and all other models. We note here that ALT is a model which combines a thin lithosphere and a constant viscosity mantle with a recent ice history, that was derived with a different solid Earth model (Lambeck et al.,1998).



Normalized Scalar Product of 3-D displacements for VM4 and the other models

The spatial pattern of the scalar product reveals some spatial pattern in the correlation between model pairs. To the right, the normalized scalar product is shown for pairs of VM4 and the other models. For the pair (VM4,VM2), the scalar product is close to 1 everywhere except for some areas around the former Fennoscandian ice sheet. The only difference between these two models is in the viscosity of the upper mantle, and this difference appears to affect the present-day signal in Europe more than in North America. For the pair (VM4,JXM), for large areas the scalar product is close to 1, while areas with large deviations between the two models are found in northern Eurasia, Greenland, part of Antarctica, Australia and the Western coast of North America. The main difference between these two models is in the ice history. For the pair (VM4,REF), an additional area of disagreement is found for most of the oceanic area between 60°S and the equator. REF and ALT use basically the same ice history (ICE-3G) and have only slight differences in the mantle viscosity structure. Finally, for (VM4,ALT), most of the ocean areas show a scalar product much less than 1 and for most parts negative, except for the central northern Pacific.

PGR and Rigid Plate Rotations

Plate	Model	Angular velocity (°/Ma) Ω_x	Ω_y	Ω_z
EU	**Original data – No PGR**	**-0.030 ±0.001**	**-0.137 ±0.001**	**0.208 ±0.001**
	Scaled VM2	-0.017	-0.134	0.209
	Scaled VM4	-0.025	-0.134	0.211
	Scaled REF	-0.031	-0.133	0.206
	Scaled ALT	-0.032	-0.134	0.205
	Scaled JXM	-0.031	-0.136	0.205
NA	**Original data No PGR**	**0.020± 0.0004**	**-0.193 +0.002**	**-0.018 ±0.002**
	Scaled VM4	0.021	-0.186	-0.011
	Scaled REF	0.015	-0.190	-0.013
	Scaled ALT	0.017	-0.188	-0.013
	Scaled JXM	0.019	-0.191	-0.012
	SNREF	*0.018*	*-0.185*	*-0.024*
SA	**Original data – No PGR**	**0.059 ±0.003**	**0.079 ±0.004**	**0.043 ±0.002**
	Scaled REF	-0.069	-0.085	-0.038
	Scaled ALT	-0.069	-0.085	-0.038
AU	**Original data – No PGR**	**0.413 ±0.002**	**0.320 ±0.002**	**0.328 ±0.002**
	Scaled VM2	0.419	0.316	0.333
	Scaled VM4	0.417	0.315	0.333
	Scaled ALT	0.417	0.315	0.331

Cartesian coordinates of angular velocity. Results are shown when using original ITRF2000 data, as well as for those models where adding a scaled pgs velocity to the translated GPS velocities leads to a significantly improved data fit when calculating rigid body rotation.

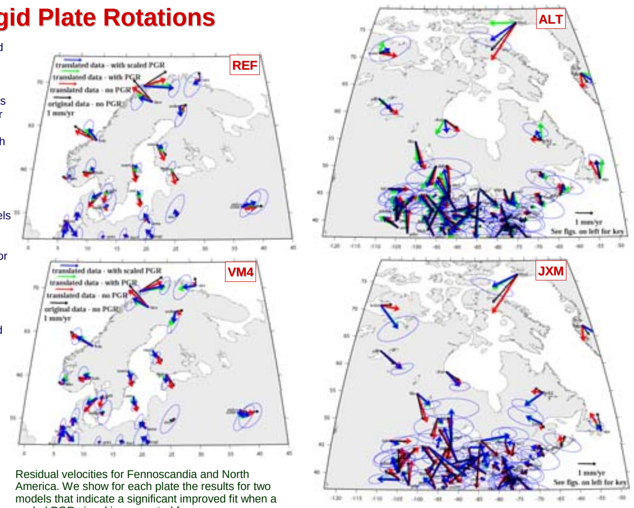
Here we calculate rigid body rotation parameters (table left) and reduced chi-squared statistics (table below). We either do this using the original ITRF2000 horizontal velocities, the translated velocities (for each PGR model used in the translation), and the translated velocities minus the PGR prediction, where we consider a case with using the PGR estimates directly and a case where we solve for a additional scaling parameter for the PGR predictions when solving for the best fitting angular velocity as well. This modeling allows us to see which PGR model is consistent with the data, and also to observe whether angular velocities will change significantly if PGR signal is accounted for.

We apply an F-test to verify whether the improved fit for the case with a scaled PGR signal subtracted is significant over a model without PGR. We find the fit significant improved for several plates and for many models (table below). However, these improvements do suggest a significant scaling of the PGR signal and it being different for different plates. Moreover, some scaling factors are near zero or negative (VM models for NA and all for AU), indicating a possible deficiency in these PGR predictions there.

Plots of residual velocities (right) indicate that residuals within PGR affected regions are smaller when PGR is taken into account, but remaining residuals still show a PGR fingerprint (see EU) or remain hard to interpret (see NA)

Plate	original data – no PGR	VM2 No PGR	VM2 With PGR	VM2 γ	VM4 No PGR	VM4 With PGR	VM4 γ	REF No PGR	REF With PGR	REF γ	ALT No PGR	ALT With PGR	ALT γ	JXM No PGR	JXM With PGR	JXM γ
EU	2.16	2.29	2.24/ 2.04	0.52 ±0.10	2.28	2.07/ 2.06	0.73 ±0.14	2.38	1.81/ 1.78	1.37 ±0.11	2.33	1.94/ 1.91	1.51 ±0.16	2.31	2.18/ 1.81	0.54 ±0.05
NA	2.34	2.58	7.26/ 2.59	-0.03 ±0.01	2.52	6.77/ 2.41	-0.21 ±0.02	2.94	2.20/ 2.15	1.40 ±0.12	2.72	2.70/ 2.61	0.53 ±0.06	2.82	1.61/ 1.61	1.08 ±0.06
SA	0.85	0.79	0.84/ 0.81	-0.14 ±0.58	0.78	0.78/ 0.79	0.44 ±0.86	0.81	0.70/ 0.71	0.81 ±0.31	0.83	0.76/ 0.71	3.15 ±1.21	0.83	0.78/ 0.80	1.00 ±0.50
AU	3.56	2.43	4.16/ 1.93	-0.99 ±0.25	2.47	3.71/ 1.88	-1.40 ±0.34	2.18	2.49/2 .19	-0.61 ±0.40	2.37	2.40/ 2.43	-1.48 ±1.88	2.38	3.25/ 2.04	-1.29 ±0.42

Reduced χ^2 values from fit between horizontal velocities and rigid body rotation. We consider several cases: using the original ITRF2000 data (second column), using the translated ITRF2000 velocities (using each PGR model), and using those translated velocities minus the PGR prediction of that model. As an alternative model we add a parameter γ to scale the PGR correction in order to find the best-fitting rigid plate motion when considering the PGR data (χ^2 for those models shows as second value in "With PGR" columns. Because we introduce an extra model parameters for the latter models we assess with an F-test whether considering the scaled PGR data leads to a significant improved fit within 95% confidence. If so (and even with a possible



Residual velocities for Fennoscandia and North America. We show for each plate the results for two models that indicate a significant improved fit when a scaled PGR signal is accounted for.

Conclusions

Regional inter-model differences in predictions of the present-day 3-D velocity field of the Earth's surface due to PGR are found to be larger than the uncertainties in the observed velocity field, particularly for the formerly ice-covered regions in North America and Eurasia. Consequently, space-geodetic observations provide valuable constraints for these models. As a main result of our validation study we find the predictions based on the ICE-5G history inconsistent with the observed velocity field in North America.

Accounting for the PRG signal in the determination of the rigid body rotation improves the estimates for the two plates with the largest deloading of former ice loads, i.e., North America and Eurasia, while for plates in the far-field of the former ice loads, the improvement is either small or negligible. In these regions, the PGR signal may be below the error of the observed velocity field or erroneous for several reasons (including the effect of lateral heterogeneities in the solid Earth).

Acknowledgements

The authors thank J. X. Mitrovica, W.R. Peltier, H.H.A Schotman, L.L.A. Vermeersen and J. van Hove for making the PGR predictions available.

We also thank all members of the IGS community and in particular the Scripps global IGS analysi centers and the IGS regional associate analysis centers for Australia, Europe, Japan, and North- and South America.

Contact:

Corné Kreemer, Nevada Bureau of Mines and Geology, University of Nevada, Mailstop 178, Reno. Email : kreemer@unr.edu ; Phone : 775-784-6691x154

References

Davies, P., and G. Blewitt (2000), Methodology for global geodetic time series estimation: A new tool for geodynamics, *J. Geohpys. Res.*, *105*, 11,083-011,100.

Lambeck, K., Smither, C., Johnston, P., 1998: Sea-level change, glacial rebound and mantle viscosity of northern Europe, Geophys. J. Int., 134, 102-144.

Milne, G. A., Mitrovica, J. X., and Davis, J. L., 1999. Near-field hydro isostasy: the implementation of a revised sea-level equation, Geophys. J. Int., 139, 464-482.

Peltier, W. R., 2004. Global glacial isostasy and the surface of the ice age Earth: The ICE-5G(VM2) model and GRACE, Ann. Rev. Earth Planet Sci., 32, 111-149.

Peltier, W. R., 2005. Description of the submission to the IERS GGFC SBL call. Available at http://www.sbl.statkart.no/projects/pgs/.

Schotman, H.H.A., Vermeersen, L.L.A, and van Hove, J.: Description of theDEOS submission to the IERS GGFC SBL call. Available at http://www.sbl.statkart.no/projects/pgs/.