The Concepts in Sea Level Studies: A Detailed Analysis of Errors

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Introduction

In the context of climate change and global warming, sea level has gained a central role for two reasons, namely (1) as a quantity potentially constraining the mass balance in the global water cycle, and (2) as a quantity associated with one of the main impacts of a global warming potentially threatening a large part of the human population. The impact-related quantity is Local Sea Level (LSL), while for the mass balance the global ocean mass is a key indicator. Estimates of a Global Sea Level Rise (GSLR) over the last 100 years have been utilized in the climate change discussion as an indication of changes in the ocean volume and mass. Scenarios of future global and regional sea level changes are increasingly used to plan mitigation in order to protect the densely populated coastal areas. However, the discussion of sea level rise and the mass balance of the global water cycle, as well as the impact of sea level changes is hampered by the absence of a well defined terminology that is based on a commonly accepted basic concept reflecting the underlying physics properly.

Here, a physically motivated set of terms is defined and the relations between these terms are specified. Then, these terms are used to study the error budget of global changes in sea level and to discuss concequences for a sea level observation system.

Definitions

Local Sea Level (LSL): Local height h of the ocean surface above the underlying solid Earth, i.e.

$$h(\lambda, \theta, t) = \begin{cases} r_1(\lambda, \theta, t) - r_0(\lambda, \theta, t) & : \text{ in the ocean} \\ 0 & : \text{ on land} \end{cases} ,$$

where r_0, r_1 are the geocentric positions of the sea floor and sea level, respectively (Plag, 2006b), and λ, θ geographical longitude and latitude, respectively. Note that h is an absolute quantity, independent of the reference frame used for r_0 and r_1.

Sea Surface Height (SSH): Local height H of the sea surface with respect to the origin of a geocentric reference frame (geocentric SSH) or an ellipsoid (ellipsoidal SSH). This quantity is a relative (i.e. reference-frame dependent) quantity, which needs the frame information attached to it.

Global ocean volume: $V_O = \int_O \mathrm{d}V = \int_0^{2\pi} \int_0^{\pi} \left(\int_{r_0(\lambda,\vartheta)}^{r_1(\lambda,\vartheta)} r^2 dr \right) \sin\vartheta \mathrm{d}\vartheta \mathrm{d}\lambda$, with ϑ co-latitude.

Global ocean mass: $M_O = \int_O \rho \mathrm{d}V$, with ρ: density.

V_O and M_O are absolute quantities. Note that r_0, r_1, and ρ are functions of time.

Relations between LSL, V_O, and M_O

M_O is an important quantity in the mass balance equation of the global water cycle. This equation can be written as

$$0 = \sum_{i=1}^{n} \frac{\mathrm{d}M_i}{\mathrm{d}t} = \sum_{i=1}^{n} \dot{M}_i ,$$

where M_i is the total mass of the water in reservoir i, and n the number of reservoirs in the global water cycle. Changes in M_O are independent of where the mass fluxes come from or go to.

However, there is no simple equation relating changes in V_O or LSL to changes in the global water cycle. V_O is a complex function of M_O, the heat and salinity content of the ocean, and the distribution of the mass in the ocean. Changes in V_O result from steric changes (i.e. changes caused by temperature and salinity changes of the sea water) and mass changes (due to mass added to or subtracted from the ocean). Moreover, the density ρ of the sea water is a non-linear function of temperature, salinity and pressure (see e.g. Gill, 1982). Consequently, V_O depends not only on the amount but also the distribution of heat, mass and salinity in the ocean.

The relation between mass transport in the global water cycle and LSL is given by the so-called sea level equation (first published by Farrell & Clark, 1976, and later extended by others):

$$\xi(\vartheta, \lambda, t) = c(t) + O(\vartheta, \lambda, t) \int_{-\infty}^{t} \int_0^{\pi} \int_0^{2\pi} G(\vartheta, \lambda, \vartheta', \lambda', t - t') \cdot$$

$$\frac{\mathrm{d}}{\mathrm{d}t'} \left\{ O(\vartheta', \lambda', t') \rho_W \xi(\vartheta', \lambda, t') + \left[1 - O(\vartheta', \lambda', t') \right] \rho_L \eta(\vartheta', \lambda, t') \right\} \sin\vartheta' \mathrm{d}\lambda' \mathrm{d}\vartheta' \mathrm{d}t' .$$

where ξ is the change in LSL (i.e. with respect to the deformable surface of the solid Earth), ϑ, λ, and t are co-latitude, longitude, and time, respectively, G is the Green's function for LSL, O the ocean function (which is 1 over the ocean and 0 over land), η the cumulated water or ice load change due to mass added to or removed from land, ρ_W and ρ_L are the densities of the ocean water and the load on land (water or ice), respectively, and $c(t)$ is a quantity included to ensure mass conservation. Mass-induced LSL changes depend on where mass has been relocated.

The LSL Equation

We denote low-frequency LSL variations at a point $\vec{x} = (\lambda, \theta)$ on the Earth surface by $\xi_{1f}(\vec{x}, t)$ (understanding that these variations are given with respect to an appropriate mean $h_0(\vec{x})$), where t is time. ξ_{1f} can be approximated as a sum of several factors as listed on the right.

All these contributions depend on the location, and most of them display a wide range of spatial and temporal scales (for a detailed discussion, see e.g. Emery & Aubrey, 1991; Plag, 2006a,b). Moreover, the local contributions to LSL from the mass-dependent terms are described by the sea level equation given above and depend not only on local processes but rather the mass transport in the global water cycle.

The LSL equation is used as a tool to set up a detailed error budget of local, regional and global estimates of sea level changes obtained from tide gauge and satellite altimetry measurements. Moreover, it is demonstrated that the LSL equation provides a basis for setting up local scenarios of plausible future LSL variations needed for the planning of mitigation and adaptation. The equation emphasizes the need to establish such local scenarios in a global geodetic reference frame, putting high demands on the accuracy of this frame.

Low-frequecy LSL variations are caused by changes in a number of processes:

$\xi_{1f}(\vec{x}, t)$	
$= T(\vec{x}, t)$	Long-period tides
$+S(\vec{x}, t)$	Steric volume changes
$+C(\vec{x}, t)$	Ocean currents
$+F(\vec{x}, t)$	Ocean freshening
$+A(\vec{x}, t)$	Atmospheric circulation
$+I(\vec{x}, t)$	Mass of large ice sheets
$+G(\vec{x}, t)$	Mass of glaciers
$+L(\vec{x}, t)$	Land hydrosphere
$+P(\vec{x}, t)$	Postglacial rebound
$+V(\vec{x}, t)$	Vertical land motion.

(Plag, 2006b)

Critical comments on widely used terms

LSL as defined above is often referred to as *Relative Sea Level*, particularly in connection with tide gauge observations. However, properly maintained tide gauges measure LSL (which is an absolute quantity) directly. Therefore, relative sea level is consider a misnomer and not used here. Likewise, the terms 'geocentric sea level' or 'absolute sea level', which are often used to denote the geocentric sea surface position as measured by satellite altimetry, are considered misnomers and are not used here. Changes in geocentric or ellipsoidal SSH only coincide with LSL changes if the ocean floor at this position does not move vertically and if the reference frame is realized with the origin in the Center of Mass of the Earth system (CM). Therefore, SSH changes are normally not equivalent to LSL changes.

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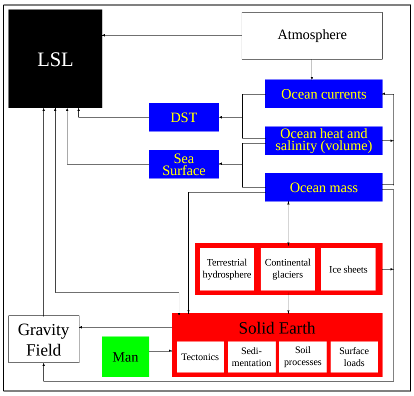

Figure 1: Processes affecting LSL. At any location, LSL is the output of Earth system processes acting on a wide range of spatial and temporal scales. For low frequencies, this leads to a complex equation of LSL as a function of the heat and salinity distribution in the ocean, ocean currents and atmospheric circulation, in the large ice sheets, continental glaciers, and the terrestrial hydrosphere, postglacial rebound, endogenic and anthropogenic vertical land motion and geoid changes, as well as changes in shape and extent of the ocean basins. The fingerprint of all mass-related contributions (Plag & Jüttner, 2001) are described by the so-called 'sea level equation', which links mass movements to sea level changes. Figure from Plag (2006b).

Errors in global LSL averages

The global tide gauge network samples LSL rather incomplete. A two degree grid derived from the tide gauges covers less than 5% of the global ocean surface (Plag, 2006a). Due to their geographical distribution, the tide gauges significantly oversample and undersample any Antarctic and Greenland contributions, respectively, and sense an apparent negative contribution of PGR (Table 1). However, they appear to sense a steric signal comparable to the global average.

Global average LSL changes determined from tide gauges are bound to be biased significantly if the contribution from mass exchanges are not modeled and extrapolated onto a global grid (see Plag, 2006a, for details).

Table 1: Bias in global averages determined from tide gauges observations.

Contribution	TG Signal
PGR	-0.1 to -0.2 mm/yr
Antarctica	+230%
Greenland	+45%
Steric	100-120%

Errors in global SSH averages

(a) Drift of reference-frame origin: Error $\delta\dot{H}$ due to a trend $\vec{V}$ between reference-frame origin and CM:

$$\delta\dot{H} = -0.08813 V_x - 0.04836 V_y - 0.09326 V_z$$

For the region covered by T/P:

$$\delta\dot{H} = -0.09389 V_x - 0.04847 V_y - 0.11536 V_z .$$

(b) Scale error in reference frame: error in global mean SSH equals scale error.

Errors in converting SSH to LSL

(c) Postglacial rebound: mean vertical motion of ocean bottom is of the order of 0.35 mm/yr.

(d) Current mass movements: mean vertical motion of ocean bottom is of the order of 0.1 mm/yr.

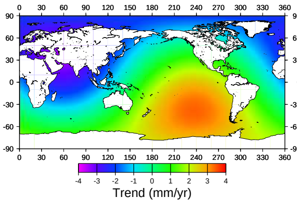

Figure 2: Effect of differential motion between ITRF origin and the CM on local vertical motion. The vertical motion is given for a differential velocity of $\vec{V} = (-1.5, -2.2, -2.1)$ mm/yr, which was estimated by Kierulf & Plag (2006) as the geocenter velocity seen by GPS alone with respect to ITRF2000. The mean vertical motion over the complete surface of the ocean is 0.43 mm/yr, and over the surface covered by Topex/Poseidon 0.49 mm/yr.

Future LSL scenarios

For future LSL scenarios, in addition to the trend in ξ_{1f}, also changes in the high-frequency part ξ_{hf} need to be considered. The latter is given by

$$\xi_{hf}(t) = \xi_{waves}(t) + \xi_{tidal}(t) + \xi_{atmos}(t) + \xi_{seiches}(t) + \xi_{tsunami}(t).$$

The LSL equation given on the left is not directly related to observations. For local LSL studies and scenarios, the low-frequency part of LSL can be viewed, in principle, as the sum of four terms: (1) local changes in the volume of the sea water due to temperature and salinity changes, (2) local changes in the sea surface due to mass exchanges in the global water cycle, (3) vertical motion of the land with respect to the center of mass of the Earth system, (4) changes in the LSL due to changing atmospheric forcing (air pressure, wind, evaporation, precipitation and radiation). This leads to a simplified LSL equation of the form

$$\xi(t) = \xi_{steric}(t) + \xi_{mass} - \xi_{land} + \xi_{atmosphere}.$$

This equation can be used to interpret observed past LSL changes as well as to set up future LSL scenarios (see Plag et al., 2006).

Consequences for the sea level observation system

Carefully maintained tide gauges, for which any relative movement of the tide gauge with respect to the underlying land is known, measure absolute LSL changes directly. Therefore, tide gauges are the corner stone of the sea level observing system, and a dense global network of well maintained long-term stable tide gauges network is crucial for measuring LSL and deriving unbiased global averages.

Observations of vertical land motion are required in order to separate the various processes contributing to LSL and to set up future LSL scenarios. Gravity satellite missions are a crucial component in the observing system for LSL variations on regional to global scale.

Satellite altimetry, which measures the reference-frame dependent position of the sea surface, is a valuable component of the sea level observing system, if appropriately integrated with tide gauges and models. The latter are required to predict the vertical motion of the ocean bottom, so that SSH observations can be correctly transformed into LSL. Thus, in order to derive changes in LSL from satellite altimetry observations, the equation

$$\xi(t) = \delta H(t) + \delta r_0(t)$$

can be used, where $\delta H(t)$ is the observed sea surface variation and $\delta r_0(t)$ the modelled vertical motion of the ocean bottom. Care need to be taken that $\delta H(T)$ and $\delta r_0(t)$ are given in the same reference frame to avoid errors described above.

Conclusions

LSL defined as the local distance from ocean bottom to sea surface is an absolute quantity, i.e. reference-frame independent. LSL is directly related to the volume of the ocean and, for known density distributions, the mass of the ocean. At coastal locations, appropriately established and operated tide gauges measure LSL directly. However, the global tide gauge network poorly samples the spatial pattern in LSL and models are required to extrapolate any signal captured by the tide gauges to global averages.

Correcting tide gauge observations of LSL for local vertical land motion does not lead to 'sea level' or 'absolute sea level changes' but to a different reference-frame dependent quantity, which is no longer directly related to ocean volume changes and which is not easy to interpret or model.

Satellite altimetry does not measure LSL but rather the geocentric position of the sea surface denoted here as SSH. The latter is reference-frame dependent and not directly related to ocean volume and mass changes. Global averages determined from SSH observations cannot be interpreted in terms of ocean volume and mass changes without modelling the physical processes forcing LSL and SSH changes. The latter requires knowledge of the mass transport in the global water cycle, in particular the geographical location of the sources of water added to the ocean. In one sentence: understanding and quantifying global and local sea level changes requires the understanding and quantifying of the mass transport in the global water cycle.

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